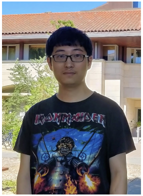
Provably Efficient Reinforcement Learning with Linear Function Approximation under Adaptivity Constraints

Tianhao Wang[1]



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NeurIPS 2021

Outline

Motivation: adaptivity constraints in Reinforcement Learning

Problem setting

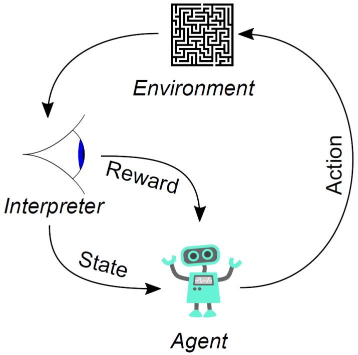
Main results: algorithm and analysis

Numerical experiment

Conclusion

(Online) Reinforcement Learning

In online Reinforcement Learning (RL), one of the most important tasks is to learn the optimal policy which maximizes the long-term cumulative rewards:



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- The limited adaptivity setting has been studied for many online learning scenarios including PFE (Kalai and Vempala, 2005), MAB (Arora et al., 2012), etc.
- A similar concept is known as *low switching cost* in RL (Bai et al., 2019), but the goal there is to achieve $\tilde{O}(\sqrt{K})$ regret with as few policy switches as possible

Our setting: limited number of policy updates

Given the number of episodes K, assume that there is a hard budget B on the number of policy switches:

$$\sum_{k=1}^{K-1} \mathbb{1}\{\pi^k \neq \pi^{k+1}\} \leq B$$

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- Batch learning model: policy switches only happen at the prefixed grids $1 = t_1 < \cdots < t_B < t_{B+1} = K + 1$

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We study the above two models in the context of linear MDPs, beyond tabular MDPs studied in Bai et al. (2019)

Linear Markov Decision Process (MDP)

We consider the setting of linear MDP (Yang and Wang, 2019; Jin et al., 2020) where both the transition probabilities and reward functions can be linearly parametrized as

$$\mathbb{P}_h(s'|s,a) = \langle \boldsymbol{\phi}(s,a), \boldsymbol{\mu}_h(s') \rangle, \;\; r_h(s,a) = \langle \boldsymbol{\phi}(s,a), \boldsymbol{\theta}_h \rangle.$$

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- The action-value function $Q_h^\pi(s,a)$ is also linear in the feature mapping ϕ (Jin et al., 2020), i.e., $\exists \mathrm{w}_h^\pi$ s.t.

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- We adapt the original LSVI-UCB algorithm (Jin et al., 2020) to allow for adaptivity constraints

Batch learning model: `LSVI-UCB-Batch`

Algorithm 1 LSVI-UCB-Batch

Set $b \leftarrow 1,\ t_i \leftarrow (i-1)\lfloor \frac{K}{B} \rfloor + 1, i \in [B]$ (uniform batch grids)
for episode $k = 1, 2, \ldots, K$ **do**
    if $k = t_b$ (time to switch the policy) **then**
        $b \leftarrow b + 1,\ Q^k_{H+1}(\cdot, \cdot) \leftarrow 0$
        Compute optimistic estimates $\{Q^k_h\}$ by backward regression
        Update the greedy policy π^k induced by $\{Q^k_h\}_{h \in [H]}$
    else
        $\pi^k \leftarrow \pi^{k-1}$ (keep the current policy)
    end if
    Run policy π^k to obtain the trajectory $\{(s^k_h, a^k_h, r_h(s^k_h, a^k_h))\}$
end for

- A batched version of the original LSVI-UCB (Jin et al., 2020)

Regret of LSVI-UCB-Batch

Theorem (W., Zhou, Gu)

Under technical assumptions and with appropriate choice of parameters, the total regret of LSVI-UCB-Batch is bounded by

$$\mathit{Regret}(T) \leq \tilde{O}\left(dHT/B + \sqrt{d^3 H^3 T}\right).$$

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- Can we do better?
- Yes, by using adaptive batch size

Rare policy switch model: `LSVI-UCB-RareSwitch`

Algorithm 2 LSVI-UCB-RareSwitch

Initialize $\mathbf{\Lambda}_h = \mathbf{\Lambda}_h^0 = \lambda \mathbf{I}_d$ for all $h \in [H]$
for episode $k = 1, 2, \ldots, K$ **do**
    $\mathbf{\Lambda}_h^k \leftarrow \sum_{\tau=1}^{k-1} \phi(s_h^\tau, a_h^\tau)\phi(s_h^\tau, a_h^\tau)^\top + \lambda \mathbf{I}_d$ (covariance matrix)
    if $\exists h, \det(\mathbf{\Lambda}_h^k) > \eta \det(\mathbf{\Lambda}_h)$ (trigger policy switch) **then**
        $\{\mathbf{\Lambda}_h\} \leftarrow \{\mathbf{\Lambda}_h^k\}$ (maintain the last covariance matrix)
        Compute optimistic estimates $\{Q_h^k\}$ by backward regression, update the corresponding greedy policy π^k
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Algorithm 3 LSVI-UCB-RareSwitch

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Initialize Λ_h = Λ_h^0 = λI_d for all h ∈ [H]
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- Related to the doubling trick (Jaksch et al., 2010; Abbasi-Yadkori et al., 2011; Zhou et al., 2021)
- The policy switch slows down as k grows

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Under technical assumptions and with appropriate choice of parameters, the total regret of `LSVI-UCB-RareSwitch` *satisfies*

$$\mathit{Regret}(T) \le \tilde{O}\left(\sqrt{d^3 H^3 T[1 + T/(dH)]^{dH/B}}\right).$$

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- Trade-off between the total regret bound and the number of policy switches
- When choosing η to be a constant (or equivalently, $B = \Omega(\log T)$), `LSVI-UCB-RareSwitch` reduces to the algorithm studied in Gao et al. (2021)

Numerical experiments

We examine the performance of our algorithms on a hard-to-learn linear MDP instance (Zhou et al., 2021) with $K = 2500$

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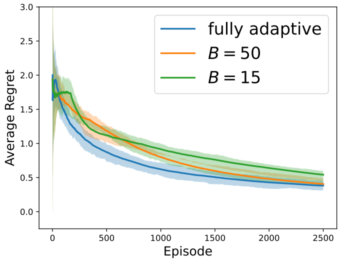
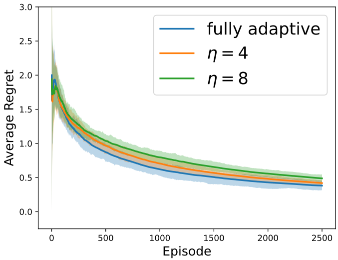


LSVI-UCB-Batch



LSVI-UCB-RareSwitch

Plot of average regret, Regret$(T)/K$, v.s. the number of episodes. The results are averaged over 50 rounds of each algorithm, and the error bars are the $[20\%, 80\%]$ empirical confidence intervals.

Conclusion

- We study episodic linear MDP under adaptivity constraints
- For the batch learning model, we propose `LSVI-UCB-Batch` which achieves a $\tilde{O}(\sqrt{d^3H^3T} + dHT/B)$ regret (the dependency on B is tight due to a complimentary lower bound)
- For the rare policy switch model, we propose `LSVI-UCB-RareSwitch` which achieves a $\tilde{O}(\sqrt{d^3H^3T[1 + T/(dH)]^{dH/B}})$ regret
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Thank you!

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